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Arima, A; Iachello, F

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ELEMENTARY EXCITATIONS IN VIBRATIONAL NUCLEI

A. ARIMA

Department of Physics, Faculty of Science, University of Tokyo, Tokyo, Japan

and

F. IACHELLO

Kernfysisch Versneller Instituut, University of Groningen, Groningen, The Netherlands

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We propose to describe the entire collective spectrum of vibrational nuclei in terms of few *interacting* elementary excitation modes. We discuss in detail the case in which only two elementary modes are important (the quadrupole d and the octupole f-bosons). We give explicit expressions for the energy levels and transition matrix elements.

The collective states of vibrational nuclei have escaped up to now a simple (but detailed) description similar in spirit to that of rotational nuclei. Many models [1, 2] have been proposed but these are either too complicated to allow a direct comparison with experiment or too simple to account for a quantitative description of the observed properties.

Recently, following a suggestion of Feshbach and Iachello [3], we have shown [4] that a *quantitative* description of the properties of some of the positive parity states in vibrational nuclei can be achieved in terms of a gas of *interacting* quadrupole bosons (hereafter called d-bosons). The fact that the quadrupole boson 2^+ is the basic excitation mode of vibrational nuclei has been known for quite some time [2, 5]. However, by exploiting the property that the states of n d-bosons form the basis for the totally symmetric irreducible representations of the five dimensional special unitary group $SU(5)$ and of its subgroup $O(5)$ we have been able to determine analytically the eigenvalues of the interacting boson Hamiltonian. They are given by

$$E(n, v, n_\Delta, L, M) = \epsilon n + \alpha \frac{n(n-1)}{2} + \beta(n-v)(n+v+3) + \gamma[L(L+1) - 6n] \quad (1)$$

where ϵ is the energy of the d-boson, α, β, γ are three parameters which describe the boson-boson interaction and n, v, n_Δ, L, M are the quantum numbers which completely label the representations of $O(5)$ [4].

It appears now that an equally simple and detailed description of the states not included in eq. (1) (for instance the negative parity states) is possible in terms of few, elementary excitation modes. We thus propose that *all* states of collective character be described in vibrational nuclei in terms of a set of independent but *interacting* excitation modes. In even-even nuclei these modes are boson modes. Since they are supposed to be independent, the operators b_{lm}^\dagger, b_{lm} which create and annihilate different excitations commute

$$[b_{l'm'}^\dagger, b_{lm}^\dagger] = \delta_{ll'} \delta_{mm'} \quad (2)$$

In terms of these operators the interacting boson Hamiltonian may be written as

$$H = \sum_{lm} \epsilon_l b_{lm}^\dagger b_{lm} + \sum_{\lambda, l'l''l'''} c_{\lambda, l'l''l'''} [(b_l^\dagger b_l^\dagger)_\mu^\lambda (b_{l''} b_{l'''})_\mu^\lambda]_0^0 \quad (3)$$

where the ϵ_l are the free boson energies, and the coefficients $c_{\lambda, l'l''l'''}$ describe the boson-boson interaction. The parenthesis in this equation denote angular momentum couplings. To the same order of approximation the transition operators Q_m^l may be written as

$$Q_m^I = q_I (b_{Im}^\dagger + (-)^{I-m} b_{I,-m}) + \sum_{l'l''} t_{l,l'l''} [b_{l''}^\dagger b_{l'}]_m^I. \quad (4)$$

The model of ref. (4) is a special case of eqs. (3) and (4). These two equations describe vibrational nuclei in the same sense in which the shell-model describes a set of *interacting* fermion single-particle degrees of freedom. Eq. (3) is the equivalent of the shell-model Hamiltonian, and eq. (4) that of transition operators. The only difference lies on the first term in the right-hand side of eq. (4) which is absent in the fermion case since the transition operator has to conserve in that case the particle number.

Despite their being very general eqs. (3) and (4) generate specific "band" structure, a band being defined as *a set of levels connected by strong E2 transitions* [4]. They also yield simple expressions for the energies of the members of the bands as well as for intraband and extraband transition matrix elements. To show this we turn to a specific example. We consider the case in which only two degrees of freedom are important, the quadrupole d-boson and the octupole 3^- boson, hereafter called f. The states of the system are of the form $d^n \otimes f^m$. Of particular importance is the set of states $d^n \otimes f$, since these states can decay through the second term in eq. (4) by E1 transitions to the states of the d^{n+1} configurations, which, as shown in ref. (4), form the ground state bands of vibrational and quasi-vibrational nuclei. Therefore these states can and have been detected in coincidence experiments with the ground state band. Using eq. (3) it is relatively simple to show that the diagonal matrix elements of H for the $d^n \otimes f$ configurations are given by

$$\begin{aligned} \langle d^n(J_1)f;I|H|d^n(J_1)f;I \rangle \\ = E[d^n(J_1)] + \epsilon_3 + n(2J_1 + 1) \sum_J [d^{n-1}(J)dJ_1|] d^n J_1]^2 \sum_{J'} (2J' + 1) c_{J'}^{(23)} \left\{ \begin{matrix} J & 2J_1 \\ 3 & I & J' \end{matrix} \right\}^2 \end{aligned} \quad (5)$$

where

$$c_{J'}^{(23)} \equiv \langle dfJ'|V|dfJ' \rangle \equiv c_{J', dfdf}, \quad (6)$$

are the matrix elements of the boson-boson interaction, eq. (3), and the other symbols have obvious meaning. The energies $E[d^n(J_1)]$ are given by eq. (1). Similarly the off-diagonal matrix elements are given by

$$\begin{aligned} \langle d^n(J_1)f;I|H|d^n(J'_1)f;I \rangle = n(2J_1 + 1)^{1/2} (2J'_1 + 1)^{1/2} \sum_J [d^{n-1}(J)dJ_1|] d^n J_1] \\ \times [d^{n-1}(J)dJ'_1|] d^n J'_1] \cdot \sum_{J'} (2J' + 1) c_{J'}^{(23)} \left\{ \begin{matrix} J & 2J_1 \\ 3 & I & J' \end{matrix} \right\} \left\{ \begin{matrix} J & 2J'_1 \\ 3 & I & J' \end{matrix} \right\}. \end{aligned} \quad (7)$$

These expressions simplify further in the case in which the states of the d^n configuration to which the f-boson is coupled belong to the ground state band $J_1 = 2n$. Then eq. (5) becomes

$$\langle d^n(J_1 = 2n)f;I|H|d^n(J_1 = 2n)f;I \rangle = E[d^n(J_1 = 2n)] + \epsilon_3 + n(2J_1 + 1) \sum_{J'} (2J' + 1) c_{J'}^{(23)} \left\{ \begin{matrix} J_1 - 2 & 2 & J_1 \\ 3 & I & J' \end{matrix} \right\}^2. \quad (8)$$

In the case in which the total spin I corresponds to the totally aligned value $I = 2n + 3$ and to the totally aligned value minus one, $I = 2n + 2$, even simpler expressions can be obtained. For these states the Hamiltonian of eq. (3) is already diagonal and its eigenvalues are given by

$$E[d^n(J_1 = 2n)f;I = 2n + 3] = E[d^n(J_1 = 2n)] + \epsilon_3 + n c_5^{(23)} \quad (9)$$

and

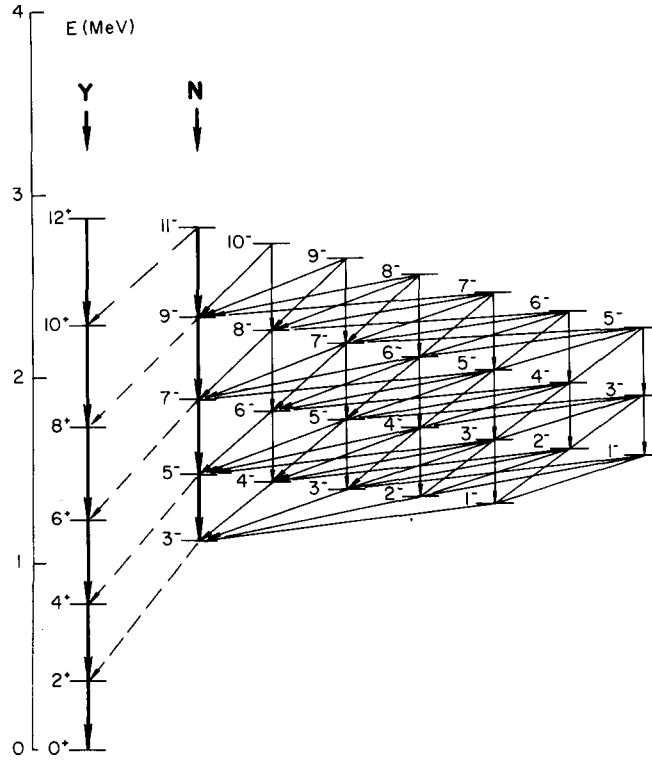


Fig. 1. Typical spectrum generated by eq. (7). Full lines represent E2 transitions, broken lines E1 transitions. The ground state and the totally aligned negative parity band are labeled Y and N respectively. For the sake of clarity only E1 transitions between these two bands are shown in the figure. The parameters used are: $\epsilon_2 = 365$ keV, $c_4^{(23)} = 45.3$ keV; $\epsilon_3 = 1123$ keV, $c_5^{(23)} = -10$ keV, $c_4^{(23)} = -50$ keV, $c_3^{(23)} = -90$ keV, $c_2^{(23)} = -130$ keV, $c_1^{(23)} = -130$ keV, $c_1^{(23)} = -170$ keV.

$$E[d^n(J_1 = 2n)f; I = 2n + 2] = E[d^n(J_1 = 2n)] + \epsilon_3 + nc_5^{(23)} + \frac{2}{5}n\Delta_4^{(23)} + \frac{3}{5}\Delta_4^{(23)} \quad (10)$$

where

$$\Delta_4^{(23)} = c_4^{(23)} - c_5^{(23)}. \quad (11)$$

In fig. 1 we show the corresponding spectrum. The negative parity states to the right of the totally aligned (N) band and the totally aligned minus one band do not form a band structure since they are admixed through eq. (7) to other states. In the figure only the diagonal part of their energy is shown. The totally aligned states are likely to be experimentally observed since they are connected by stretched E2 transitions. An application of the linear relationship, eq. (9), to ^{152}Gd is shown in fig. 2. This nucleus is not completely vibrational but its ground state band can still be described by the boson formula (4)

$$E[d^n(J_1 = 2n)] = \epsilon_2 n + c_4^{(22)} \frac{n(n-1)}{2}. \quad (12)$$

Fits of the same quality have been performed [7] to the odd parity levels in ^{150}Sm , ^{154}Dy and ^{156}Er with $c_5^{(23)} = -47, -33$ and -184 keV, respectively, and a r.m.s. deviation, defined as

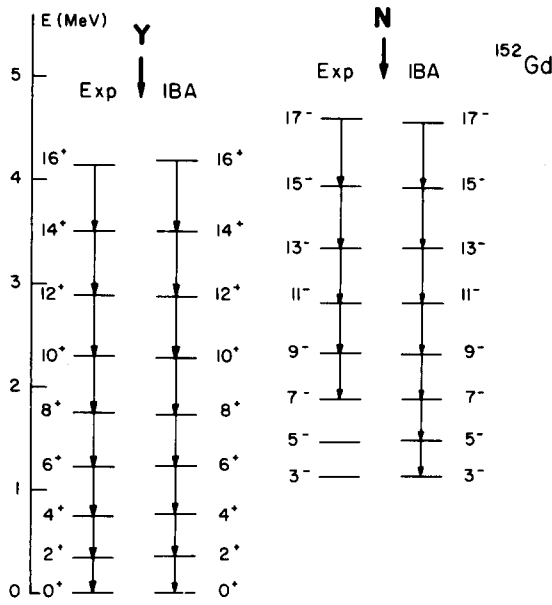


Fig. 2. Comparison between experimental [6] and theoretical (IBA) energies of the ground state (Y) and negative parity (N) band in ^{152}Gd . The parameters in the theoretical spectrum are $\epsilon_2 = 365$ keV, $c_4^{(22)} = 45.3$ keV; $\epsilon_3 = 1123$ keV, $c_5^{(23)} = -10$ keV.

$$\text{r.m.s.} = \left[\sum_{i=1}^K \frac{(E_i^{\text{th}} - E_i^{\text{exp}})^2}{K-2} \right]^{1/2} \quad (13)$$

of 29 keV (^{150}Sm), 19 keV (^{154}Dy) and 60 keV (^{156}Er). In eq. (13) K is the total number of levels fitted (9, 5 and 8, respectively).

Intraband and extraband transition matrix elements may also be simply calculated within the framework of the interacting boson model. The intraband matrix elements are all given in terms of the number q_2 . For instance

$$B[E2(d^{n+1}f J' = 2n+5) \rightarrow (d^n f J = 2n+3)] = (n+1)q_2^2$$

$$B[E2(d^{n+1}f J' = 2n+4) \rightarrow (d^n f J = 2n+3)] = (n+1) \frac{3}{(2n+3)(n+1)} q_2^2 \quad (14)$$

$$B[E2(d^{n+1}f J' = 2n+4) \rightarrow (d^n f J = 2n+2)] = (n+1) \frac{n}{n+1} \frac{2n+5}{2n+3} q_2^2.$$

The extraband transition matrix elements are instead given in terms of the numbers $t_{l, df}$. For instance

$$B[E1(d^n f J' = 2n+3) \rightarrow (d^{n+1} J = 2n+2)] = (n+1)t_{1,23}^2. \quad (15)$$

From eqs. (14) and (15) one can obtain the branching ratio

$$\frac{B[E1; (I = 2n+3)^- \rightarrow (I = 2n+2)^+]}{B[E2; (I = 2n+3)^- \rightarrow (I = 2n+1)^-]} = \frac{n+1}{n} c \quad (16)$$

where $c = t_{1,23}^2/q_2^2$ is a constant for each nucleus.

Results similar to those presented here for the configuration $d^n \otimes f$ may be obtained for the coupling of other elementary excitations, be that a collective boson state, a two-quasiparticle state or a single quasiparticle state in odd A nuclei. In particular the totally aligned state of any configuration $d^n \otimes j$ satisfy the simple linear relation

$$E[d^n(2n)j; I = 2n+j] = E[d^n(2n)] + E(j) + n c_{d+j}^{(dj)}. \quad (17)$$

It is worth mentioning that a number of coupled bands of the type just described have been recently observed in the light Er isotopes by the Brookhaven group [8].

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